Maximum Power Point Tracking using the Optimal Duty Ratio for DC-DC Converters and Load Matching in Photovoltaic Applications

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***Abstract*-The purpose of this paper is to present an alternative maximum power point tracking, MPPT, algorithm for a photovoltaic module, PVM, to produce the maximum power, P_{max}, using the optimal duty ratio, *D*, for different types of dc-dc converters and load matching. The proposed algorithm has the advantages of maximizing the efficiency of the power utilization, can be integrated to other MPPT algorithms without affecting the PVM performance, is excellent for Real-Time applications and is a robust analytical method, different from the traditional MPPT algorithms which are more based on trial and error, or comparisons between present and past states. The procedure to calculate the optimal duty ratio for a buck, boost and buck-boost converters, to transfer the maximum power from a PVM to a load, is presented in the paper. Additionally, the existence and uniqueness of optimal internal impedance, to transfer the maximum power from a photovoltaic module using load matching, is proved. Finally, results are presented in the paper.**

I. Introduction

Solar energy is one of the most important alternatives energies with applications in urban areas, motor drives, satellites, [1]-[8] etc. A photovoltaic module, PVM, is the key component to convert solar energy into electric energy [1]. In addition to the PVM, a typical dc photovoltaic system configuration consists of storage capacitance, dc-dc converter, and batteries [2]. In most of the applications, it is always desired to obtain the maximum power from a PVM, due the fact that the PVM operates at the highest efficiency [3]-[4]. The maximum power point tracker, MPPT, is the typical algorithm to calculate the maximum power, P_{max}, provided by a PVM [3]-[6].

In the past, many authors described different variations of the MPPT algorithm, [3]-[12] and the applications to control dc-dc converters in energy conversion, [8]-[14]. Unfortunately, most of the existing MPPT methods to estimate the maximum power are based on trial and error algorithms where the voltage is increased until the maximum power is achieved, better known as the hill-climbing method [5]-[7]. Other MPPT algorithms compare the last sampled voltage and current versus the presently sampled voltage and current to see which state will produce the maximum power [11]. Additionally, the literature offers other types of MPPT algorithms such that rippled based method [7], fuzzy logic [9] and look-up table methods [10].

Disadvantages with these MPPT algorithms are that discrete algorithms require several iterations to calculate the optimal steady-state duty ratio [13]. Some of them are not designed for quick changes in the weather conditions [13]. Also, for non-analytical methods, the time for the iterations will depend on the initial conditions and can create bifurcation problems [14]-[16]. In this paper, an analytical method for load matching is proposed using the optimal duty ratio for a dc-dc converter to transfer the maximum power to the load. For load matching, the internal resistance *Ri* for a PVM is used. The paper is divided into five sections which are the PVM model, optimal duty ratio for a dc-dc converter, simulations and experimental results, and conclusions.

II. Analytical Photovoltaic Module Model

The PVM model based on the manufacturer data sheets [17] will be used to obtain the optimal duty ratio, *D*. The PVM model takes into consideration the temperature, *T*, and effective irradiance, E_i, over the PVM and the Standard Test Conditions, STC, i.e. T_N is 25 °C and E_{iN} is 1000 W/m^2. The manufacturer data sheet will provide the temperature constant for the voltage, *TCV*, the temperature constant for the current, *TCi*, the open circuit voltage under STC, V_{oc}, short circuit current under STC, I_{sc}, and the PVM characteristic constant, *b*. Also, most of the manufacturers will provide the open circuit voltage, V_{max}, when E_i is more than 1200 W/m^2 and *T* is 25 °C and the open circuit voltage, V_{min}, when E_i is less than 200 W/m^2 and *T* is 25 °C [17]. V_{max} is approximately $1.03 \cdot V_{oc}$ and V_{min} is approximately $0.85 \cdot V_{oc}$. This model considers the useful data given by the manufacturer while no additional parameters are required, i.e. thermal voltage, diode reverse saturation current, band gap for the material, etc. Additionally, the PVM model is continuous and differentiable with respect to the voltage. The static PVM model is given in (1). The open circuit voltage at any *T* or E_i, *Vx* is given by (2) and is calculated when the current of operation is zero. *Ix*, the short circuit current at any *T* or E_i, is calculated when the voltage of operation is zero and is given by (2).

$$I(V)=\frac{E_i}{E_{iN}}\cdot\left(I_{sc}+TCi\cdot\left(T-T_N\right)\right)\cdot\frac{1}{1-\exp\left(-1/b\right)}\cdot\left[1-\exp\left(\frac{V}{b\cdot\frac{E_{iN}}{E_i}\cdot TCV\cdot\left(T-T_N\right)+V_{\max}-\left(V_{\max}-V_{\min}\right)\cdot\exp\left(\frac{E_i}{E_{iN}}\cdot\ln\left(\frac{V_{\max}-V_{oc}}{V_{\max}-V_{\min}}\right)\right)}-\frac{1}{b}\right)\right] \quad (1)$$

After substituting (2) and (3) into (1), a simplified PVM model is obtained and is given in (4). The PVM power is described by (5). Finally, the PVM internal resistance, *Ri*, is calculated by dividing the input voltage, *V*, by the current, *I(V)*, and is given by (6). The PVM internal resistance, *Ri*, will be used for the load matching. Typically, the batteries have an internal resistance between 0.2-0.7Ω [18] and a short circuit could be very dangerous for the battery. The PVM internal resistance is much larger and the value depends on the voltage and power drawn from the PVM. Also, a PVM is a current limited system hence can be short-circuited without damage at difference of the batteries. Finally, if the *Ri* is equal to the load resistance then P_{max} can be transfer to the load but if both resistances differ the power will be less than P_{max} [18].

$$Vx=\frac{E_{iN}}{E_i}\cdot TCV\cdot\left(T-T_N\right)+V_{\max}-\left(V_{\max}-V_{\min}\right)\cdot\exp\left(\frac{E_i}{E_{iN}}\cdot\ln\left(\frac{V_{\max}-V_{oc}}{V_{\max}-V_{\min}}\right)\right) \quad (2)$$

$$Ix=\frac{E_i}{E_{iN}}\cdot\left(I_{sc}+TCi\cdot\left(T-T_N\right)\right) \quad (3)$$

$$I(V)=\frac{Ix}{1-\exp\left(-1/b\right)}\cdot\left[1-\exp\left(\frac{V}{b\cdot Vx}-\frac{1}{b}\right)\right] \quad (4)$$

$$P(V)=\frac{V\cdot Ix}{1-\exp\left(-1/b\right)}\cdot\left[1-\exp\left(\frac{V}{b\cdot Vx}-\frac{1}{b}\right)\right] \quad (5)$$

$$Ri(V)=\frac{V-V\cdot\exp\left(-1/b\right)}{Ix\cdot\left[1-\exp\left(\frac{V}{b\cdot Vx}-\frac{1}{b}\right)\right]} \quad (6)$$

Figures 1, 2 and 3 show the P-V, R-V and I-V curves for a PVM SX-10 [20] and their relationship between the internal resistance, optimal voltage and maximum power. Figure 1 shows that there is a unique maximum point for the maximum power which will be produced when the PVM voltage is equal to the optimal voltage, *Vop*, which is unique. Then, using the fact that P_{max} is produced when the PVM voltage is equal to *Vop*, and because the derivative of the resistance with respect to the voltage is always positive, then it can be seen that there is a unique optimal internal resistance, *Rop*. So if a PVM is operating at *Vop* then the PVM will transfer P_{max} to the load. Finally, with this fact in mind, the next section will describe how to analytically calculate the optimal duty ratio for a dc-dc converter to transfer the maximum power to the load.

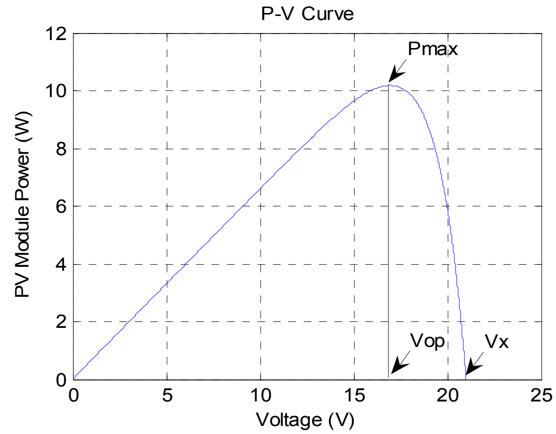

Fig. 1 *P-V* curve for the SX-10 and their relationship between P_{max}, and *Vop*.

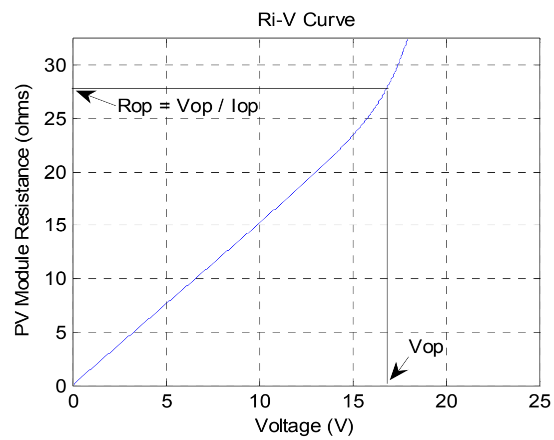

Fig. 2 *Ri-V* curve for the SX-10 and their relationship between *Vop* and *Rop*.

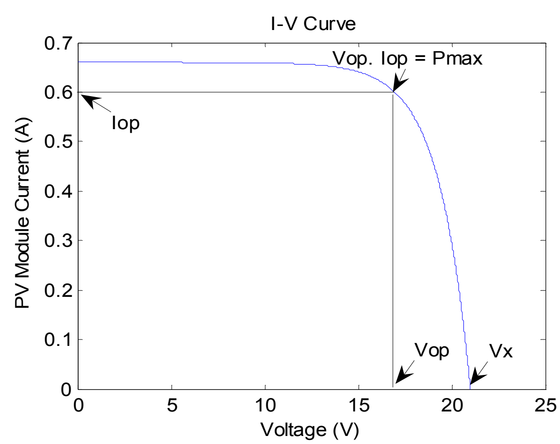

Fig. 3 *I-V* curve for the SX-10 and their relationship between *Vop* and *Iop*.

III. Optimal Duty Ratio for a dc-dc Converter to Obtain the Maximum Power

Consider a PVM connected to a buck-boost converter to supply power to a resistive load. The objective is to calculate the optimal duty ratio, *D*, so the PVM will supply P_{max}. The analysis will be done using the steady-state conditions for a buck-boost converter, where all the components are ideal, the inductor current is continuous, the capacitor is large enough to assume a constant output voltage and the switch is closed for time *D/f* and open for *(1-D)/f*, [21]. An advantage of the buck-boost converter is that the magnitude of the output voltage can be either greater than or less than the source voltage, depending on the duty ratio of the switch [21], making it excellent for photovoltaic applications where the weather conditions are changing very fast. The only minor disadvantage for the buck boost converter is the polarity reversal on the output.

The first step for load matching will be done using the relationship between the voltage input and output for a buck-boost converter relationship. The load resistance *Ro* can be seen as voltage output, *Vo*, divided by current output, Io. Using the last information, the relationship between the input resistance, *Ri*, and the output resistance, *Ro*, is given by (7). If *V* is *Vop*, hence *Ri* is *Rop*, the optimal duty cycle, *D*, can be solved. The optimal duty ratio, *D*, is obtained and only depends on *Ro* and *Rop*. Switching at the optimal duty ratio guarantees that the power supplied to load is P_{max}.

$$Ro=\frac{Vo}{Io}=\frac{-D\cdot Vi}{(1-D)\cdot Io}=\frac{D^2\cdot Vi}{(1-D)^2\cdot Ii}$$
$$=\frac{D^2\cdot Vi\cdot\left(1-\exp\left(\frac{-1}{b}\right)\right)}{(1-D)^2\cdot Ix\cdot\left[1-\exp\left(\frac{Vi}{b\cdot Vx}-\frac{1}{b}\right)\right]}=\frac{D^2\cdot Ri}{(1-D)^2} \tag{7}$$

Using (7), the optimal duty ratio, *D*, as a relationship of the optimal resistance, *Rop*, and output resistance, *Ro*, can be solved and is given in (8). Additionally, if the power input and the power output are equal to P_{max}, *D* can be expressed as a relationship between the optimal voltage, *Vop*, and the output voltage, *Vo*, as given by (9).

$$D=\frac{\sqrt{Ro}}{\sqrt{Ro}+\sqrt{Rop}} \tag{8}$$

$$D=\frac{Vo}{Vo+Vop} \tag{9}$$

The minimum inductance $L_{1\min}$ for the buck-boost converter to preserve the continuous current mode is given in (10). The voltage output ripple is calculated in (11). The same type of procedure is done to calculate the duty cycle for the buck converter or boost converter.

$$L_{1\min}=\frac{R\cdot(1-D)^2}{2\cdot f}=\frac{Ro\cdot Rop}{2\cdot f\cdot\left(\sqrt{Ro}+\sqrt{Rop}\right)^2}<\frac{Ro}{2\cdot f} \tag{10}$$

$$Voripple=\frac{D}{f\cdot C_1\cdot Ro}=\frac{1}{C_1\cdot f\cdot\left(Ro+\sqrt{RoRop}\right)} \tag{11}$$

Table 1 shows the conditions and optimal duty ratio for a buck converter, boost converter and buck-boost converter. Form Table 1, the only disadvantage of using a buck or boost converter is the restriction in the values of *Rop* and *Ro* for both cases.

TABLE I
Optimal *D* for Different dc-dc Converters for Load Matching

DC-DC converter	*D* for any *Po*	*D* when $Pi=Po=P_{max}$	Required
Buck-Boost	$D=\frac{\sqrt{Ro}}{\sqrt{Ro}+\sqrt{Ri}}$	$D=\frac{Vo}{Vo+Vop}$	None
Boost	$D=1-\sqrt{\frac{Ri}{Ro}}$	$D=1-\frac{Vop}{Vo}$	$Ro>Rop$
Buck	$D=\sqrt{\frac{Ro}{Ri}}$	$D=\frac{Vo}{Vop}$	$Rop>Ro$

Finally, this method for load matching can be integrated to other algorithms such that the linear reoriented coordinates method, LRCM, which is described in details in [22]. The LRCM is an analytical method used to calculate *Vop* and *Iop*, then P_{max} is calculated. Using the LRCM, the optimal resistance, *Rop*, is calculated under any changes in *T* or *Ei* and is given in (12). Also, *Rop* can be calculated using *Ix* and *Vx*, then the optimal duty ratio is calculated and used on the dc-dc converter to transfer P_{max} from the PVM to the load.

$$Rop=\frac{Vop}{Iop}=\frac{\left[\frac{E_{iN}}{E_i}\cdot TCV\cdot(T-T_N)+V_{\max}-(V_{\max}-V_{\min})\cdot\exp\left(\frac{E_i}{E_{iN}}\cdot\ln\left(\frac{V_{\max}-V_{oc}}{V_{\max}-V_{\min}}\right)\right)\right]\cdot\left(1+b\cdot\ln\left(b-b\cdot\exp\left(\frac{-1}{b}\right)\right)\right)\cdot\left(1-\exp\left(\frac{-1}{b}\right)\right)}{\left[\frac{E_i}{E_{iN}}\cdot\left(I_{sc}+TCi\cdot(T-T_N)\right)\cdot\frac{1}{1-\exp\left(-1/b\right)}\right]\cdot\left(1-b+b\cdot\exp\left(\frac{-1}{b}\right)\right)} \tag{12}$$

IV. Results

Figure 4 present the algorithm to validate and test the proposed load matching method. Figure 5 shows an integrated PV system using a pyranometer to measure the irradiance level and thermocouples to measure the temperature over the PVM surface. The integrated PV system has a Sharp ND-208U1 PVM [23] with P_{max} is 208 W, *Rop* is 2.65 Ω, *Vop* is 23.48 V, *Ix* is 0.75 A, *Vx* is 30 V and *b* is 0.1, connected to a dc bus with capacitance 400 µF. The dc-dc converter is a 50 kHz buck-boost converter with inductance 100 µH and capacitance 400 µF, and the resistive load is 0.75 Ω. The objective of the presented PV system is to supply 208W (i.e. the maximum power) produced by the PVM to the resistive load. Figure 6 shows the transient results simulations for the integrated PV system and the simulations were done using Simulink. These results show how a PVM can be controlled and deliver P_{max} using the proposed load matching strategy.

V. Conclusions

In this paper new contributions to the field of solar energy conversion were presented. The first contribution is the use of the optimal duty ratio and load matching to transfer the maximum power of the PVM to the load. The proposed method can be integrated with other algorithms such as LRCM to calculate the PVM internal resistance. Also, since the derivative of *Ri(V)* with respect to the voltage is positive, existence and uniqueness was proven, and the optimal internal resistance, *Rop*, which will transfer the maximum power to the load was calculated. Also, the optimal duty ratios for different types of dc-dc converters for a PVM to supply P_{max} were derived. Finally, the method is extremely efficient, easy to program in a DSP and applicable to other nonlinear power sources such as fuel cells.

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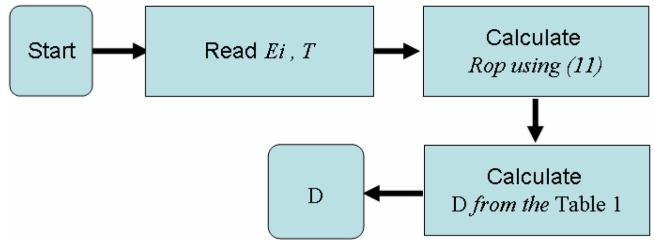

Fig. 4 Algorithm to calculate the optimal duty ratio given E_i and T.

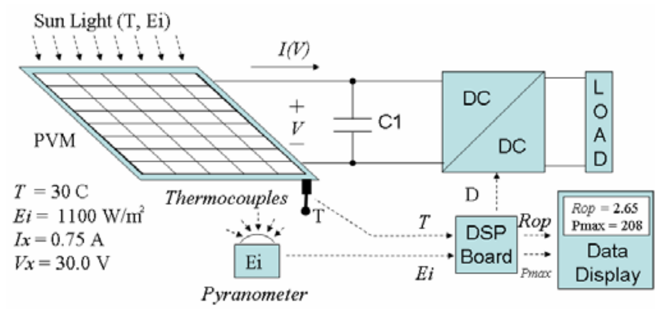

Fig. 5 Integrated PV system using load matching and the optimal duty ratio given *Ei* and *T*.

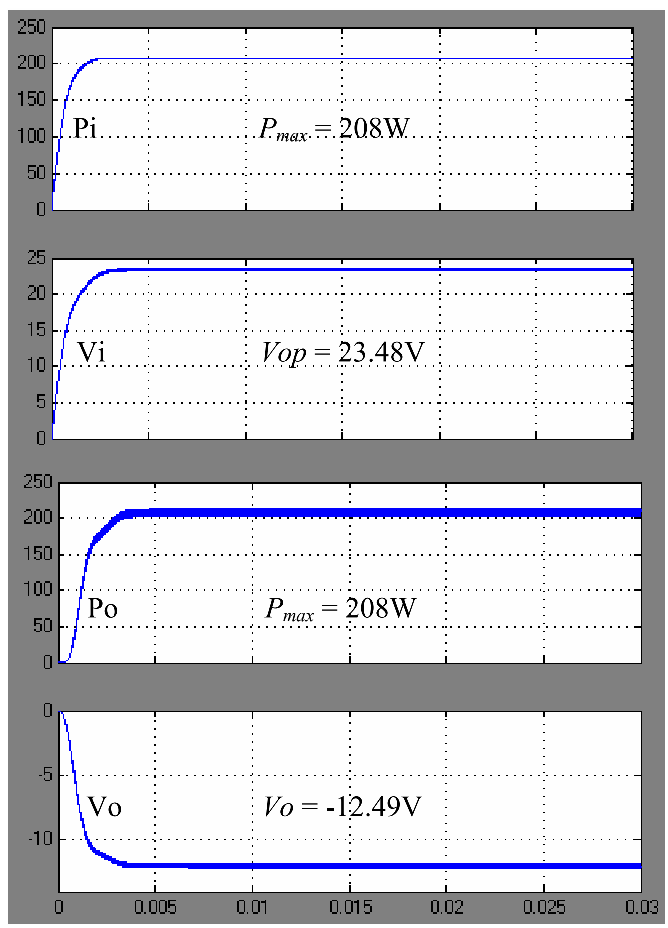

Fig. 6 Power supplied by the PVM, voltage supplied by the PVM, Buck-Boost converter power output, and Buck-Boost converter voltage output.

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